

## Self-Attention

input vectors  $X: N_k \times D_k$

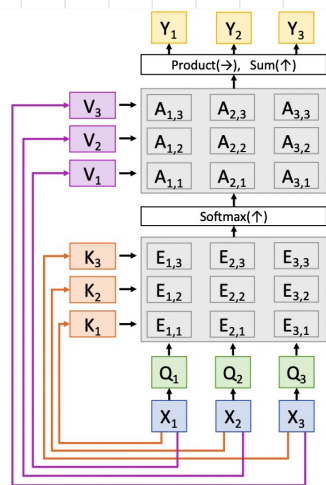
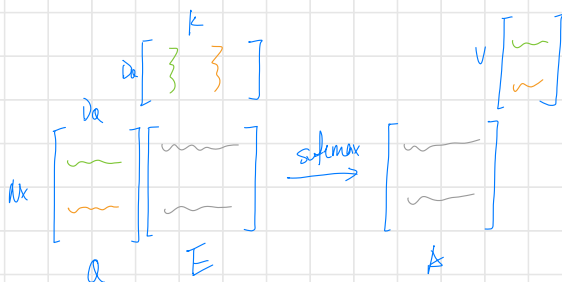
$N_k$  is the sequence length, not batch size

key matrix  $W_k: D_k \times D_k$

value matrix  $W_v: D_k \times D_v$

query matrix  $W_q: D_k \times D_q$

$$\left. \begin{array}{l} \text{query: } Q = X \cdot W_q \\ \text{key: } K = X \cdot W_k \\ \text{value: } V = X \cdot W_v \end{array} \right\} \begin{array}{l} \text{similarities } E = QK^T / \sqrt{D_k} \\ A = \text{softmax}(E, \text{dim}=1) \end{array}$$



self-attention is permutation equivariant

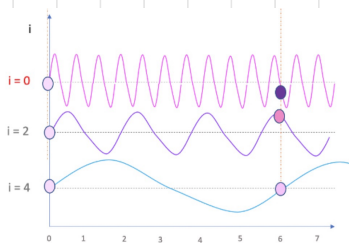
## Positional Encoding

requirement: the encoding is deterministic  
 output a unique encoding for each time-step  
 distance between any two time-steps should be consistent across sequences of different lengths

let  $E_t \in \mathbb{R}^d$  be the positional encoding at time-step  $t$

$$E_{2t}[t] = \sin\left(\frac{t}{10000^{2t/d}}\right)$$

$$E_{2t+1}[t] = \cos\left(\frac{t}{10000^{2t/d}}\right)$$



$$\begin{bmatrix} E_{2k}[t+\phi] \\ E_{2k+1}[t+\phi] \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{t+\phi}{10000^{2k/d}}\right) \\ \sin\left(\frac{t+\phi}{10000^{2k/d}}\right) \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{\phi}{10000^{2k/d}}\right) & -\sin\left(\frac{\phi}{10000^{2k/d}}\right) \\ \sin\left(\frac{\phi}{10000^{2k/d}}\right) & \cos\left(\frac{\phi}{10000^{2k/d}}\right) \end{bmatrix} \cdot \begin{bmatrix} \cos\left(\frac{t}{10000^{2k/d}}\right) \\ \sin\left(\frac{t}{10000^{2k/d}}\right) \end{bmatrix}$$

$$\left[ \begin{array}{c} \text{~~~~~} \\ \text{~~~~~} \\ \text{~~~~~} \\ \text{~~~~~} \end{array} \right] \rightarrow e^{i\theta t} \quad \theta = \frac{1}{10000^{2k/d}}$$

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