$$\min_{x} \ \max_{i} \ Ax + b \qquad \text{non differentiable}$$

$$\Downarrow$$

$$\min_{x,t} \ t \qquad \text{inequality constrained}$$
$$\text{s.t.} \ Ax + b \le t \cdot \mathbb{1}$$

$$\Downarrow$$

$$\min_{x,t} \ c \cdot t - \sum \log(t \cdot \mathbb{1} - Ax - b) \qquad \text{log-barrier}$$

$$\Downarrow$$

$$\min. \ f(x,t) = [0 \ \ c] \cdot \begin{bmatrix} x \\ t \end{bmatrix} - \sum \log\left([-A \ \ \mathbb{1}] \begin{bmatrix} x \\ t \end{bmatrix} - b \right)$$

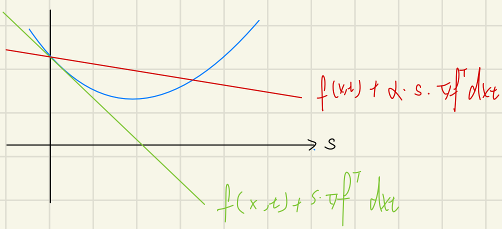
$$\nabla_{x,t} f(x,t) = [0 \ \ c] - [-A \ \ \mathbb{1}]^T \cdot \ 1/\left([-A \ \ \mathbb{1}] \cdot \begin{bmatrix} x \\ t \end{bmatrix} - b\right)$$

$$\nabla^2_{x,t} f(x,t) = [-A \ \ \mathbb{1}]^T \ \mathrm{diag}\left([-A \ \ \mathbb{1}] \cdot \begin{bmatrix} x \\ t \end{bmatrix} - b\right)^{-2} [-A \ \ \mathbb{1}]^T$$

- Solve log-barrier with newton method

$$\nabla_{x,t} f(x+dx, t+dt) \approx \nabla_{x,t} f(x,t) + \nabla^2_{x,t} f(x,t) \begin{bmatrix} dx \\ dt \end{bmatrix} := 0$$

$$d_{xt} = \nabla^2_{x,t} f(x,t)^{-1} \cdot -\nabla_{x,t} f(x,t)$$



- solve Piece-wise linear with Interior-point method

  solve log-barrier problem, the duality gap is given by m/c $(A \in \mathbb{R}^{m \times n})$