- Linearization

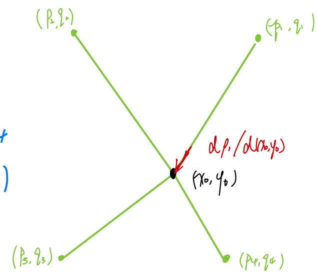
$$f(x) \approx f(x_0) + Df(x_0)(x-x_0) \qquad Df(x_0)_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{x=x_0}$$

x near x_0 $\Rightarrow$ $f(x)$ very near $f(x_0) + Df(x_0)(x-x_0)$: differentiable

x near x_0 $\Rightarrow$ $f(x)$ ~~very~~ near $f(x_0)$ $\cancel{+ Df(x_0)(x-x_0)}$: Continuous

eg. Navigation by range measurement

(x,y): unknown coordinate in plane
(p_i, q_i): known coordinate of beacons $i = 1,2,3,4$
ρ_i : measured distance from (x,y) to (p_i, q_i)



$\rho \in \mathbb{R}^4$ is a non-linear mapping : $\rho_i(x,y) = \sqrt{(x-p_i)^2 + (y-q_i)^2}$

unit vector pointing from (p_i, q_i) to (x_0, y_0)

linearize around (x_0, y_0): $\delta\rho = A \cdot \begin{bmatrix} \delta x \\ \delta y \end{bmatrix} = \begin{bmatrix} \frac{2(x_0-p_i)}{\sqrt{(x_0-p_i)^2+(y_0-q_i)^2}} & \frac{2(y_0-q_i)}{\sqrt{(x_0-p_i)^2+(y_0-q_i)^2}} \\ - & - \\ - & - \\ - & - \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$

- Matrix multiplication

$C = AB \qquad O(n^3)$ can be done in $O(n^{2.41})$

- vector space of functions

$V = \{x : \mathbb{R}_+ \to \mathbb{R}^n \mid x \text{ is differentiable}\}$, where vector sum is sum of functions

$(x+z)(t) = x(t) + z(t) \qquad (\alpha x)(t) = \alpha \cdot x(t)$

elements in V are functions, and form a subspace