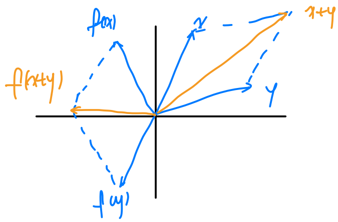
- Linear Equations

$$f(x+y) = f(x) + f(y)$$
$$f(\alpha x) = \alpha f(x)$$



- Matrix multiplication function

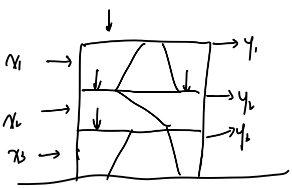
  $f(x) = Ax$ is linear

  All linear functions can be written as $f(x) = Ax$

eg. linear elastic structure

x_i is some external force along some direction

y_i is some (small) deflection of some node, along some direction

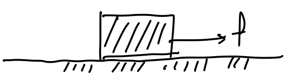


$y \approx Ax$ when x small ($A_{11} > 0$ since x_1 positively contribute to y_1)

A: compliance matrix

a_{ij} gives deflection i per unit force at j (m/N)

eg Final position/velocity of mass due to applied forces



- fractionless surface
- unit mass, 0 position/velocity at $t=0$, subject to force $f(t)$ for $0 \le t \le n$
- piece-wise constant force : $f(t) = x_j$ for $j-1 \le t < j$
- y_1, y_2 are final position and velocity
- $y = Ax$ $A \in \mathbb{R}^{2\times n}$

m/N

first row is decreasing: initial state matter more to final state

(the newton applied at $t=0$ last for n seconds)

second row are 1's 1N/(m/s)

e.g. Network traffic and flow

n flows with rate $f_1 \cdots f_n$ pass from source nodes to destination node

$$\text{edges}\updownarrow\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}\begin{bmatrix} \\ f \\ \\ \end{bmatrix} \qquad A_{ij} = 1\{\text{flow } j \text{ goes through node } i\}$$

let $d_1 \cdots d_m$ be edge delays, $l_1 \cdots l_n$ are delay for a route.

$l = A^T d$

· Linearization

$$f(x) \approx f(x_0) + Df(x_0)(x - x_0) \qquad Df(x_0)_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{x=x_0}$$

x near x_0 $\Rightarrow$ $f(x)$ very near $f(x_0) + Df(x_0)(x - x_0)$: differentiable

x near x_0 $\Rightarrow$ $f(x)$ ~~very~~ near $f(x_0) \cancel{+ Df(x_0)(x - x_0)}$: continuous