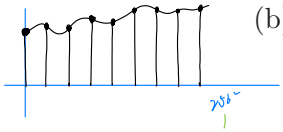
4. (15 points) *More on sampling.* Let $g(t)$ be an integrable, band-limited signal with $\mathcal{F}g(s) = 0$ for $|s| \geq p/2$. Express the integral

$$\int_{-\infty}^{\infty} g(t)\, dt$$

in terms of sample values of $g$. What is the minimum sampling rate that will allow us to do this?

5. (15 points) *Matlab Exercise on Sampling:* Obtain the image
http://see.stanford.edu/materials/lsoftaee261/man.gif

(a) Load the 8-level gray-scale 256x256 image in Matlab using 'imread' command. Convert the matrix image into a vector of length $256^2$ (using Matlab's ':' command) and call it the 'time-domain' signal $x(n)$. Change the values in $x(n)$ from type 'uint8' to 'double'. Normalize it such that the maximum signal value is 1.



(b) Find the Fourier transform $X(f)$ of $x(n)$ using the FFT command, where $X(f)$ has length the same length as $x(n)$. Plot the magnitude response of $X(f)$ centered at 0 i.e. on the X-axis, $f$ ranges from $-\frac{256^2}{2}$ to $\left(\frac{256^2}{2} - 1\right)$ (total $256^2$ points).

to sample at rate $2p$, take $2p$ points in interval $[0, 256^2 - 1]$ (see $256^2$ as 1 second)

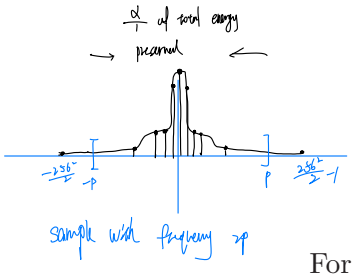
(c) Define the 'bandwidth', $N_B(\alpha)$, of the signal $x(n)$ as the frequency values that contain fraction $\alpha$ of the total energy. Mathematically, this is equivalent to

spacing $= \frac{256^2}{2p}$

$$E_{\text{total}} \triangleq \sum_{f=-\frac{256^2}{2}}^{f=\frac{256^2}{2}-1} |X(f)|^2$$

$$E_m \triangleq \sum_{f=-m}^{f=m} |X(f)|^2$$

$$N_B(\alpha) \triangleq 2p \quad \text{s.t.} \quad \frac{E_p}{E_{\text{total}}} = \alpha$$




For the following $\alpha$ values, $\alpha = 0.9, 0.95, 0.99$, sample the signal $x(n)$ at the corresponding Nyquist rate. In order to sample $x(n)$ at a rate $\frac{1}{r}$, simply pick every $r$th value in $x(n)$, so that the resulting sampled-signal $x_s(n)$ will be of length $\frac{256^2}{r}$. Find the re-constructed signal $\hat{x}(n)$ of length $256^2$ using interpolation on the sampled-signal $x_s(n)$. For each value of $\alpha$, the methods to be used for interpolation are (i) replication by nearest-neighbor and (ii) linear. In the two interpolation schemes, reconstruction of any point has contributions from 1 and 2 nearest neighbors respectively. Display the signal $\hat{x}(n)$ as a 256x256 image. Submit the original image, the 6 reconstructed images and Matlab code. Briefly explain the artifacts seen in the reconstructed images. Matlab commands

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$$

$$y = \frac{(y_2 - y_1)(x - x_1)}{(x_2 - x_1)} + y_1$$

$$= \frac{(y_2 - y_1)(x - x_1)}{\text{spacing}} + y_1$$

```
imagesc(); colormap('gray')
```

can be used to display the image.