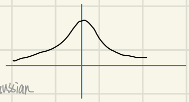
$$\mathcal{F} f(t) = \int_{-\infty}^{+\infty} f(t)\, e^{-2\pi i s t}\, dt$$
$$\mathcal{F}^{-1} g(t) = \int_{-\infty}^{+\infty} g(t)\, e^{2\pi i s t}\, dt$$
$\{s\}$ is the spectrum

different definition of F.T

$$\mathcal{F} f(s) = \int_{-\infty}^{+\infty} e^{-ist} f(t)\, dt$$
$$\mathcal{F} f(s) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ist} f(t)\, dt$$
$$\mathcal{F} f(t) = \int_{-\infty}^{+\infty} e^{ist} f(t)\, dt \qquad \mathcal{F}^{-1} f(t) = \int_{-\infty}^{+\infty} e^{-ist} f(t)\, dt$$
$\vdots$

eg. Gaussian

$f(t) = e^{-\pi t^2}$



the "π" normalizes the gaussian

$\int_{-\infty}^{+\infty} e^{-\pi t^2}\, dt = 1$

$$F(s) = \int_{-\infty}^{+\infty} e^{-\pi t^2} \cdot e^{-2\pi i s t}\, dt$$
$$F'(s) = \int_{-\infty}^{+\infty} e^{-\pi t^2} \cdot -2\pi i t \; e^{-2\pi i s t}\, dt$$
$$= i \int_{-\infty}^{+\infty} e^{-2\pi i s t} \left(-2\pi t \cdot e^{-\pi t^2}\right) dt$$
$$= i \int_{-\infty}^{+\infty} e^{-2\pi i s t}\, d\, e^{-\pi t^2}$$
$$= i \cdot \left[\underbrace{e^{-2\pi i s t}}_{\text{absolute value } 1} \cdot \underbrace{e^{-\pi t^2}}_{0 \text{ at both end}} \Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} e^{-\pi t^2}\, d\, e^{-2\pi i s t} \right]$$
$$= i \cdot -\int_{-\infty}^{+\infty} e^{-\pi t^2} \cdot -2\pi i s \cdot e^{-2\pi i s t}\, dt$$
$$= -\int_{-\infty}^{+\infty} 2\pi s \cdot e^{-2\pi i s t} \cdot e^{-\pi t^2}\, dt$$
$$= -2\pi s \int_{-\infty}^{+\infty} e^{-2\pi i s t} \cdot e^{-\pi t^2}\, dt$$
$$= -2\pi s\, F(s)$$

$$F'(s) = -2\pi s\, F(s)$$

$$F(s) = e^{-\pi s^2} \cdot F(0)$$
$$F(0) = \int_{-\infty}^{+\infty} e^{-\pi t^2} \cdot e^{-2\pi i 0 t}\, dt$$
$$= \int_{-\infty}^{+\infty} e^{-\pi t^2}\, dt = 1$$

$F(s) = e^{-\pi s^2}$

the Fourier Transform of Gaussian Function is itself

· Fourier Transform Duality

exploit the similarity between formulas of $\mathcal{F}$ and $\mathcal{F}^{-1}$

$$\mathcal{F}f(s) = \int_{-\infty}^{+\infty} e^{-2\pi i s t} f(t)\,dt$$

$$\mathcal{F}f(-s) = \int_{-\infty}^{+\infty} e^{2\pi i s t} f(t)\,dt = \mathcal{F}^{-1}f(s)$$

$$\mathcal{F}f(-s) = \mathcal{F}^{-1}f(s)$$

don't think of it as time domain vs. frequency

think of it as a mathematical operation $\quad f(t) \to \boxed{\mathcal{F}} \to \mathcal{F}f(s)$

introduce reversed signal $f^-(t) = f(-t)$

$$\mathcal{F}f(-s) = \mathcal{F}^{-1}f(s) \quad \to \quad (\mathcal{F}f)^-(s) = (\mathcal{F}^{-1}f)(s)$$

$$\mathcal{F}f^-(s) = \int_{-\infty}^{+\infty} f(-t)\, e^{-2\pi i s t}\,dt$$

$$= -\int_{t=-\infty}^{+\infty} f(-t)\, e^{2\pi i s(-t)}\,d(-t)$$

$$= -\int_{u=+\infty}^{-\infty} f(u)\, e^{2\pi i s u}\,du$$

$$= \int_{-\infty}^{+\infty} f(u)\, e^{2\pi i s u}\,du = \mathcal{F}^{-1}f(s)$$

$$\mathcal{F}(f^-)(s) = (\mathcal{F}^{-1}f)(s)$$

$$(\mathcal{F}f)^-(s) \Longleftrightarrow (\mathcal{F}^{-1}f)(s)$$

$$\mathcal{F}f^-(s)$$

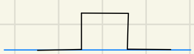
Fourier transform of reversed signal
= reversal of Fourier Transform

$$\mathcal{F}\mathcal{F}f = f^-$$

$$\mathcal{F}f = \mathcal{F}f^{--} = \mathcal{F}^{-1}f^- \qquad \mathcal{F}\mathcal{F}f = f^-$$

$$\mathcal{F}\mathcal{F}f = \mathcal{F}\mathcal{F}^{-1}f^- = f^-$$

eg. $\mathcal{F}\,\mathrm{sinc} = \mathcal{F}\mathcal{F}\,\mathrm{Rectangle}$



$= \mathrm{Rectangle}^-$

$= \mathrm{Rectangle}$

$\mathcal{F}\,\mathrm{sinc}^2 = \mathcal{F}\mathcal{F}\Lambda$

$= \Lambda^-$

$= \Lambda$