Lec 2. if $f(t) = \sum_{k=-n}^{n} C_k e^{2\pi i k t}$ $C_k = \int_0^1 f(t)\, e^{-2\pi i k t}\, dt$

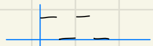
$$\int_0^1 e^{2\pi i m t} \cdot e^{-2\pi i n t}\, dt = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$$

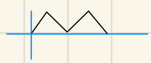
given $f(t)$ periodic, period 1.
define $\hat{f}(k) = \int_0^1 f(t)\, e^{-2\pi i k t}\, dt$

· How general

can we write $f(t) = \sum_{k=-n}^{n} \hat{f}(k)\, e^{2\pi i k t}$ for some n

can not model a $\begin{cases} \text{discontinuous} \\ \text{non differentiable} \\ \vdots \\ \text{(any discontinuity in any order derivative)} \end{cases}$ function with a finite sum





it takes high frequencies to make discontinuity
to model more general signals, we have to consider infinite sum

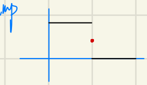
any function that's not infinitely smooth generates infinitely many Fourier coefficients

convergence of $\sum_{k=-n}^{n} \hat{f}(k)\, e^{2\pi i k t}$: "conspiracy of cancellation" to make the series converge

continuous $f(t)$: $\sum_{k=-n}^{n} \hat{f}(k)\, e^{2\pi i k t} \to f(t)$ $\forall t$ (pointwise convergence)

smooth $f(t)$: pointwise convergence
can estimate the rate of convergence over an interval depending on degree of smoothness (uniform convergence)

jump discontinuity : $\sum_{k=-n}^{n} \hat{f}(k)\, e^{2\pi i k t}$ converges to the average of the jump



general : mean square convergence
suppose $\int_0^1 |f(t)|^2\, dt < \infty$ (finite energy)
$\int_0^1 \left| \sum_{k=-n}^{n} \hat{f}(k)\, e^{2\pi i k t} - f(t) \right|^2 dt \to 0$ as $n \to \infty$