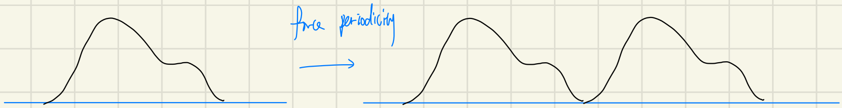


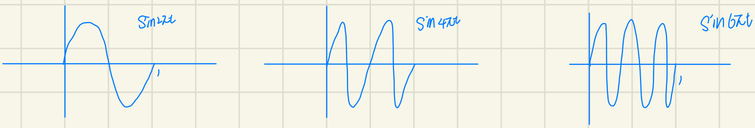
periodization of a signal

Use period 1 $\quad f(t) = f(t+1)$

model signals are $\sin 2\pi t$ and $\cos 2\pi t$ which has period 1

can modify and combine $\sin 2\pi t$ and $\cos 2\pi t$ to model general signals with period 1

1st big idea: 1 period, many frequencies



period 1, frequency 1 $\qquad$ period $\frac{1}{2}$ (1), frequency 2

sum $\quad \sin 2\pi t + \sin 4\pi t + \sin 6\pi t$

period 1, many frequencies (1, 2 and 3)

To model a complicated signal of period 1,
we can modify the amplitude, frequency and phase of $\sin 2\pi t$

$$\sum_{k=1}^{n} A_k \cdot \sin(2\pi k t + \phi_k) \qquad \text{the sum has period 1}$$

$$\sum_{k=1}^{n} A_k \sin(2\pi k t + \phi_k) \qquad \left(+ \frac{a_0}{2}\right)$$

$$= \sum_{k=1}^{n} a_k \sin(2\pi k t) + b_k \cos(2\pi k t) \qquad \left(+ \frac{a_0}{2}\right)$$

· Complex form

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \cos(2\pi k t) = \left(e^{2\pi k t} + e^{-2\pi k t}\right)/2$$

$$\sin(2\pi k t) = \left(e^{2\pi k t} - e^{-2\pi k t}\right)/2i$$

$$\sum_{k=0}^{n} a_k \sin 2\pi k t + b_k \cos 2\pi k t$$

$$= \sum_{k=0}^{n} \left[a_k \left(e^{2\pi i k t} + e^{-2\pi i k t}\right)/2 + b_k \left(e^{2\pi i k t} - e^{-2\pi i k t}\right)/2i \right]$$

$$= \sum_{k=0}^{n} \left[\left(\frac{a_k}{2} + \frac{b_k}{2i}\right) e^{2\pi i k t} + \left(\frac{a_k}{2} - \frac{b_k}{2i}\right) e^{-2\pi i k t} \right]$$

$$= \sum_{k=-n}^{n} C_k \, e^{2\pi i k t}$$

$$\left.\begin{aligned} C_k &= \frac{a_k}{2} + \frac{b_k}{2i} \\ C_{-k} &= \frac{a_k}{2} - \frac{b_k}{2i} \end{aligned}\right\} \text{symmetric coefficients} \rightarrow \text{real function value}$$

(C_0 must be real)

· How general

$f(t)$ is a periodic function with period 1.

can we write $f(t) = \sum_{k=-n}^{n} C_k \cdot e^{2\pi i k t}$

suppose we can, what happen.

$f(t) = \cdots + C_m \cdot e^{2\pi i m t} + \cdots$

$C_m \cdot e^{2\pi i m t} = f(t) - \sum_{k\neq m}^{[-n,n]} C_k \cdot e^{2\pi i k t}$

$\downarrow$ multiply by $e^{-2\pi i m t}$

$C_m = f(t) \cdot e^{-2\pi i m t} - \sum_{k\neq m}^{[-n,n]} C_k \cdot e^{2\pi i (k-m) t}$

$\downarrow$

$\int_0^1 C_m \, dt = \int_0^1 f(t)\, e^{-2\pi i m t}\, dt - \sum_{k\neq m}^{[-n,n]} C_k \int_0^1 e^{2\pi i (k-m) t}\, dt$

$\int_0^1 e^{2\pi i (k-m) t}\, dt = 0$ for $k \neq m$

(cos and sin in a period)

$\downarrow$

$C_m = \int_0^1 f(t)\, e^{-2\pi i m t}\, dt$

suppose we can write $f(t) = \sum_{k=-n}^{n} C_k\, e^{2\pi i k t}$ $\quad C_m = \int_0^1 f(t)\, e^{-2\pi i m t}\, dt$