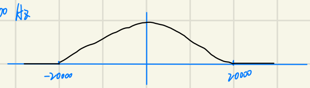
· Sample music

people can hear up to 20000 Hz

spectrum of music



$P/2 \approx 20k$ $\quad P \approx 40k$ $\quad$ Nyquist rate: 40k samples/second

then interpolate the music by $f(t) = \sum_{k=-\infty}^{+\infty} f(k/p)\, \mathrm{sinc}(pt-k)$

in fact, for CDs, 44.1 kHz

· DFT

1). find a reasonable discrete approximation of continuous signal $f(t)$

2). find a reasonable discrete approximation of $\mathcal{F}f$

3). find a reasonable of passing from discrete form of f to discrete form of $\mathcal{F}f$

Base on a mistake of sampling

(Wrongly) assume:

$f(t)$ is time-limited to $0 \le t \le L$

$(\mathcal{F}f)(s)$ is limited to $0 \le s \le 2B$

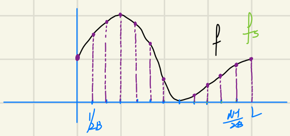
1° a signal cannot be both time-limited and frequency-limited

2° $\mathcal{F}f$ should be symmetric

(saying this to make the indexing of discrete variable easier)

According to the sampling theorem, to get an reasonable discrete approximation of f,

take samples spaced $1/2B$ apart ($t_0 = 0, \cdots t_{N-1} = N-1/2B$



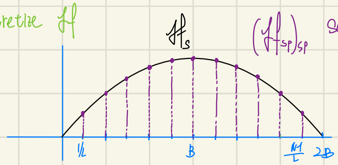
$\frac{N}{2B} = L$

discretize f

sampled form of f is: $f(t) \sum_{k=0}^{N-1} \delta_{t_k}(t) = \sum_{k=0}^{N-1} f(t_k)\, \delta_{t_k}(t)$

the Fourier Transform of $f_{sampled}$ $\quad (\mathcal{F}f_{sp})(s) = \sum_{k=0}^{N-1} f(t_k)\, e^{-2\pi i t_k s}$

discretize $\mathcal{F}f$



$(\mathcal{F}f_{sp})_{sp}$ sampled version of the F.T of sampled function

the spectrum of $\mathcal{F}f$ is limited in $[0, L]$

thus $\mathcal{F}f$ should be sampled spacing $1/L$

take samples spaced $1/L$ apart

$\frac{2B}{L} = N$ $\quad$ Also N sample points

$s_0 = 0/L$ $\quad s_{N-1} = \frac{N-1}{L}$

to think about how fast to sample a signal in the domain, should think about the range in the other domain

$$(\mathcal{F}f_{sp})_{sp}(s) = (\mathcal{F}f_{sp})(s) \cdot \sum_{m=0}^{N-1} \delta_{s_m}(s)$$

$$= \left(\sum_{k=0}^{N-1} f(t_k)\, e^{-2\pi i t_k s}\right)\left(\sum_{m=0}^{N-1} \delta_{s_m}(s)\right)$$

$$= \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} f(t_k)\, e^{-2\pi i t_k s}\, \delta_{s_m}(s)$$

$$= \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} f(t_k)\, e^{-2\pi i t_k s_m}\, \delta_{s_m}(s)$$

the sampled values of $\mathcal{F}f_{sp}$ are

$$F(s_m) = \sum_{k=0}^{N-1} f(t_k)\, e^{-2\pi i t_k s_m} \qquad n = 0 \cdots N-1$$

N sample points of Fourier Transform of the sampled function

the final step of defining DFT : eliminate the continuous part completely

$$t_k = \frac{k}{2B} \qquad s_m = \frac{m}{L} \qquad t_k \cdot s_m = \frac{mk}{2BL} = \frac{mk}{N}$$

$$e^{-2\pi i t_k s_m} = e^{-2\pi i \frac{km}{N}}$$

finally, identify that $f(t_k) = f[k]$,

likewise $F(s_m) = F[m]$

$$F[m] = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i \frac{mk}{N}} \qquad m = 0, \cdots N-1 \qquad \text{DFT}$$