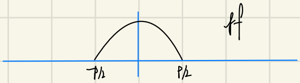
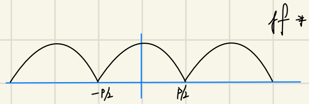




$$\Pi_p \cdot (\mathcal{F}f * Ш_p) = \mathcal{F}f$$

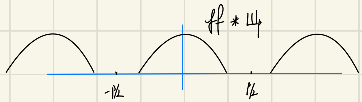
$$f(t) = \sum_{k=-\infty}^{+\infty} f(k/p)\, \text{sinc}(pt-k)$$

p: bandwidth / sampling rate (samples per second)

Nyquist rate

sample with higher rate than Nyquist rate

the derivation holds



$f(t) = \sum_{k=-\infty}^{+\infty} f(k/p)\, \text{sinc}(pt-k)$, the interpolation requires infinite number of samples

In application, need a finite sum, introduce error

a signal cannot be limited in both time and frequency

if $\mathcal{F}f(s) = 0$ for $|s| \geq p/2$, $f(t) \neq 0$ for large t

if $f(t) = 0$ for $|t| \geq q/2$, $\mathcal{F}f(s) \neq 0$ for large s

if $\mathcal{F}f(s) = 0$ for $|s| \geq p/2$

$$\mathcal{F}f \cdot \Pi_p = \mathcal{F}f$$

$$\mathcal{F}^{-1}(\mathcal{F}f \cdot \Pi_p) = f$$

$$(\mathcal{F}^{-1}\mathcal{F}f * \mathcal{F}^{-1}\Pi_p) = f$$

$$f * p\,\text{sinc}(pt) = f$$

$$(f * p\,\text{sinc}(pt))(x) = \int_{u=-\infty}^{+\infty} f(u)\, p\,\text{sinc}(p(x-u))\, du$$

$$\mathcal{F}(f*g) = \mathcal{F}f \cdot \mathcal{F}g$$

$$\mathcal{F}^{-1}(f \cdot g) = \mathcal{F}^{-1}(\mathcal{F}\mathcal{F}^{-1}f \cdot \mathcal{F}\mathcal{F}^{-1}g)$$
$$= \mathcal{F}^{-1}\mathcal{F}(\mathcal{F}^{-1}f * \mathcal{F}^{-1}g)$$
$$= \mathcal{F}^{-1}f * \mathcal{F}^{-1}g$$

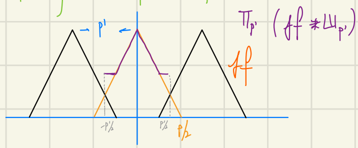
the sinc goes on forever, $f * p\,\text{sinc}(pt)$ goes on forever

clash: mathematical theorem, a signal cannot be limited in both time and frequency

In real world, signals are limited in time and frequency

· Aliasing and interpolation

what if try this for a p' that's smaller than bandwidth p (sampling rate p')



$$\mathcal{F}^{-1}[\Pi_{p'} \cdot (\mathcal{F}f * Ш_{p'})] \neq \mathcal{F}^{-1}\mathcal{F}f \qquad \Pi_{p'} \cdot (\mathcal{F}f * Ш_{p'}) \neq \mathcal{F}f$$

$$= \mathcal{F}^{-1}[\mathcal{F}\mathcal{F}^{-1}\Pi_{p'} \cdot \mathcal{F}\mathcal{F}^{-1}(\mathcal{F}f * Ш_{p'})]$$

$$= \mathcal{F}^{-1}[\mathcal{F}(\mathcal{F}^{-1}\Pi_{p'} * \mathcal{F}^{-1}(\mathcal{F}f * Ш_{p'}))]$$

$$= \mathcal{F}^{-1}\Pi_{p'} * \mathcal{F}^{-1}(\mathcal{F}f * Ш_{p'}) \qquad \mathcal{F}^{-1}(f * g) = (\mathcal{F}(f * g))^{-}$$

$$= \mathcal{F}^{-1}\Pi_{p'} * (f \cdot \mathcal{F}^{-1}Ш_{p'}) \neq f \qquad = (\mathcal{F}f \cdot \mathcal{F}g)^{-} = (\mathcal{F}f)^{-} \cdot (\mathcal{F}g)^{-} = \mathcal{F}^{-1}f \cdot \mathcal{F}^{-1}g$$

$$\vdots$$

$$= \sum_{k=-\infty}^{+\infty} f(k/p') \operatorname{sinc}(p't - k) \neq f$$

cannot get f. what do we get ??

let $g(t) = \sum_{k=-\infty}^{+\infty} f(k/p') \operatorname{sinc}(p't - k)$ $\quad$ f and g agree at sample values

$$g(m/p') = \sum_{k=-\infty}^{+\infty} f(k/p') \operatorname{sinc}\left[p'\left(\frac{m}{p'} - \frac{k}{p'}\right)\right] \qquad m \text{ is an integer, } m/p' \text{ is a sample point}$$

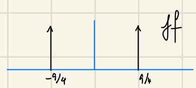
$$= \sum_{k=-\infty}^{+\infty} f(k/p') \operatorname{sinc}(m-k) \qquad \operatorname{sinc}(m-k) = \begin{cases} 1 & m=k \\ 0 & m \neq k \end{cases}$$

$$= f(m/p')$$

g is an alias of f / f and g are alias of each other

eg. $f(t) = \cos(\frac{9}{2}\pi t) = \cos(\frac{9}{4} \cdot 2\pi t)$

frequency $\frac{9}{4}$, $\mathcal{F}f = \frac{1}{2}[\delta_{9/4} + \delta_{-9/4}]$



proper sampling rate is anything greater than $9/2$

when sample with $p = 1$ $\quad \Pi_1 \cdot (\mathcal{F}f * Ш_1)$

$$\mathcal{F}f * Ш_1 = \frac{1}{2}(\delta_{\frac{9}{4}} + \delta_{-9/4}) * \sum_{k=-\infty}^{+\infty} \delta_k$$

$$= \frac{1}{2} \sum_{k=-\infty}^{+\infty} (\delta_{k+9/4} + \delta_{k-9/4})$$

$$\Pi_1 \cdot (\mathcal{F}f * Ш_1) = \frac{1}{2}(\delta_{1/4} + \delta_{-1/4})$$

$$\mathcal{F}^{-1}[\tfrac{1}{2}(\delta_{1/4} + \delta_{-1/4})](t) = \cos(\tfrac{1}{4} 2\pi t)$$

$$\left.\begin{aligned} f(t) &= \cos(\tfrac{9}{4} 2\pi t) \\ g(t) &= \cos(\tfrac{1}{4} 2\pi t) \end{aligned}\right\} \text{ agree at sample points } t = \text{integer}$$