$Ш_p(x) = \sum_{k=-\infty}^{+\infty} \delta(x-kp)$

Three important properties:

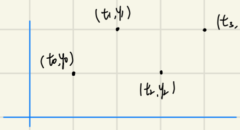
1. sampling property $f(x) \cdot Ш_p(x) = \sum_{k=-\infty}^{+\infty} f(kp)\,\delta(x-kp)$
2. periodizing property $(f * Ш_p)$ is a periodic function of period p
3. Fourier Transform property $(\mathcal{F}Ш_p)(s) = \frac{1}{p} Ш_{1/p}(s)$ $\quad (\mathcal{F}^{-1}Ш_p)(s) = \frac{1}{p} Ш_{1/p}(s)$

} swapped back and forth by taking Fourier Transform

· Interpolation problem

Interpolate all value of a function from discrete set of samples

A process evolving in time, a set of measurements at equal intervals



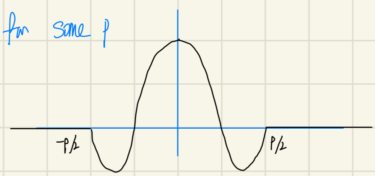
the more rapidly the function might change, the less certainty you have about the interpolation

We want to regulate how rapidly the function is oscillating between sample measurements, governed by Fourier Transform

To eliminate all frequencies beyond a certain point (by assumption).

a function is band-limited if $(\mathcal{F}f)(s) = 0$ for $|s| \geq p/2$ for some p

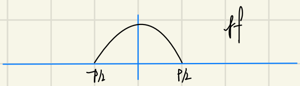
the smallest p is called band-width

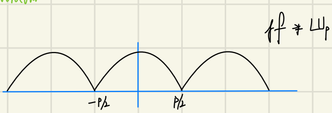


For bandlimited signals, the interpolation problem can be solved exactly

get a formula $f(t)$ for all t in terms of $y_k = f(t_k)$

Use shah function to periodize $\mathcal{F}f$ by convolution





get the original Fourier Transform by cutting off by Π_p : $\Pi_p \cdot (\mathcal{F}f * Ш_p) = \mathcal{F}f$

$$\Pi_p \cdot (\mathcal{F}f * Ш_p) = \mathcal{F}f$$

this is in the frequency domain, what happened in the time domain

$$\mathcal{F}^{-1}\left[\Pi_p \cdot (\mathcal{F}f * Ш_p)\right] = \mathcal{F}^{-1}\mathcal{F}f$$

$$\mathcal{F}\mathcal{F}^{-1}f(s) \cdot \mathcal{F}\mathcal{F}^{-1}g(s) = \mathcal{F}(\mathcal{F}^{-1}f * \mathcal{F}^{-1}g)(s)$$

$$\mathcal{F}^{-1}\left[\mathcal{F}(\mathcal{F}^{-1}\Pi_p * \mathcal{F}^{-1}(\mathcal{F}f * Ш_p))\right] = f$$

$$\mathcal{F}^{-1}\Pi_p * \mathcal{F}^{-1}(\mathcal{F}f * Ш_p) = f$$

$$(\mathcal{F}^{-1}\Pi_p)(t) = (\mathcal{F}\Pi_p)(-t) = p\,\mathrm{sinc}(pt)$$

$$\mathcal{F}^{-1}\Pi_p * (f \cdot \mathcal{F}^{-1}Ш_p) = f$$

$$\mathcal{F}^{-1}(f * g) = (\mathcal{F}(f * g))^{-}$$
$$= (\mathcal{F}f \cdot \mathcal{F}g)^{-}$$
$$= (\mathcal{F}f)^{-} \cdot (\mathcal{F}g)^{-}$$
$$= (\mathcal{F}^{-1}f \cdot \mathcal{F}^{-1}g)$$

$$[p\,\mathrm{sinc}(pt)] * \left[f(t) \cdot 1/p\, Ш_{1/p}(t)\right] = f$$

$$[p\,\mathrm{sinc}(pt)] * \left[1/p \sum_{k=-\infty}^{+\infty} f(k/p)\, \delta(t - k/p)\right] = f$$

$$\sum_{k=-\infty}^{+\infty} \mathrm{sinc}(pt) * \left(1/p\, f(k/p)\, \delta(t - k/p)\right) = f$$

$$\sum_{k=-\infty}^{+\infty} f(k/p)\left[\mathrm{sinc}(pt) * \delta(t - k/p)\right] = f$$

$$\sum_{k=-\infty}^{+\infty} f(k/p)\ \mathrm{sinc}(p(t - k/p)) = f(t)$$

if $(\mathcal{F}f)(s) = 0$ for $|s| \geq p/2$

$$f(t) = \sum_{k=-\infty}^{+\infty} f(k/p)\, \mathrm{sinc}(pt - k)$$

Nyquist Sampling formula