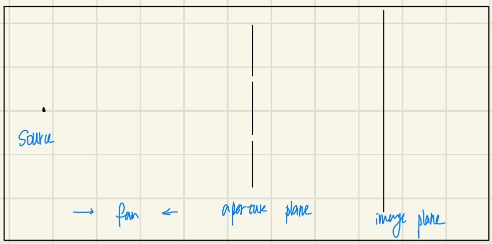
diffraction / interference patterns of light passing through apertures

- light from a distant source
- plane with apertures
- an image plane, on which diffraction pattern is shown

Assume light is
- an oscillating Electrical Magnetic (EM) field
- monochromatic (only one frequency of light that's being diffracted)

distance of the image plane determines nearfield / farfield diffraction

measure distance relative to the wavelength



far away source means that the aperture plane is a wavefront (light comes in as a plane wave)
wave have the same phase at all points of the aperture plane

represent the light on the plane by a time-oscillating electric field $\underbrace{E_\circ}_{\text{strength of the field, } E_\circ \text{ on the aperture plane}} \cdot e^{2\pi i \nu t}$ (assume the light is a single frequency)

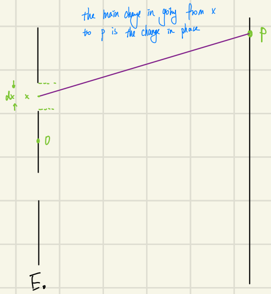
what's the electrical field at a point p on the imaging plane

the wave gets to p along different paths. how do they add up? (typically Huygens principle)

Approach with Huygens principle: every aperture can be regarded as a new source (not modern view)

the strength of the field in the hole of size dx at x $\approx E_\circ e^{2\pi i \nu t} dx$

the main change in going from x to p is the change in phase



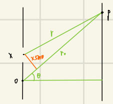
phase change in traveling a distance r from x to p: $\frac{2\pi r}{\lambda}$ where λ is the wavelength

the field at p due to field at x is $dE = E_\circ \cdot e^{2\pi i \nu t} \cdot e^{-2\pi i r/\lambda} \cdot dx$

total field: $\int_{\text{apertures}} E_\circ \cdot e^{2\pi i \nu t} \cdot e^{-2\pi i r/\lambda} \, dx$ (r depends on x)

$$= E_0 \cdot e^{2\pi i \nu t} \int_{\text{apertures}} e^{-2\pi i r/\lambda} \, dx$$

$\downarrow$ you see the magnitude of $|E|$



Fraunhoffer approximation:

when $r \gg x$, $r_0 - x\sin\theta \approx r$

(far field diffraction)

what's seen $= E_0 \int_{\text{apertures}} e^{-2\pi i r/\lambda} \, dx$

$$= E_0 \int_{\text{apertures}} e^{-2\pi i (r_0 - x\sin\theta)/\lambda} \, dx$$

$$= E_0 \, e^{-2\pi i r_0/\lambda} \int_{\text{apertures}} e^{2\pi i x \sin\theta/\lambda} \, dx$$

$p = \sin\theta/\lambda$

$$= E_0 \, e^{-2\pi i r_0/\lambda} \int_{\text{apertures}} e^{2\pi i x p} \, dx$$

introduce aperture function

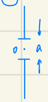
$$A(x) = \begin{cases} 1 & \text{if } x \in \text{aperture} \\ 0 & \text{if } x \text{ is opaque} \end{cases}$$

$$= E_0 \, e^{-2\pi i r_0/\lambda} \int_{-\infty}^{+\infty} A(x) \, e^{2\pi i x p} \, dx$$

$$= E_0 \, e^{-2\pi i r_0/\lambda} (\mathcal{F}^{-1} A)(p) \quad \left(= E_0 \, e^{-2\pi i r_0/\lambda} (\mathcal{F} A)(-p) \right)$$

Under far field diffraction, the intensity of light (what you see) is the magnitude of the Fourier Transform of the aperture function

eg. Single - slit diffraction



$A(x) = \Pi_a(x)$

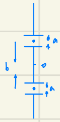
$$(\mathcal{F}A)(-p) = a \cdot \mathrm{sinc}(-ap) = a \, \mathrm{sinc}\left(\frac{a\sin\theta}{x}\right)$$

eg. delta aperture

$A(x) = \delta(x)$

$(\mathcal{F}A)(-p) = 1$ uniformly illuminated

eg. Young's double slit



$A(x) = \Pi_a(x + b/2) + \Pi_a(x - b/2)$

$\downarrow$

$a \, \mathrm{sinc}(ap) \, 2\cos(\pi b p)$ $\quad p = \sin\theta/\lambda$