Symmetric matrices

$K \in S^n$ n real eigenvalues, n orthogonal eigenvectors

$K = Q \Lambda Q^T$

eg. $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ rotation by θ

$\begin{cases} K = LDL^T & \text{LU factorization of } K \in S^n \\ K = Q\Lambda Q^T & \text{eigenvalue decomposition of } K \in S^n \end{cases}$

· Eigenvector of K

$K = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ & \ddots & \ddots & \vdots \\ & \cdots & \cdots & \end{bmatrix}$

$Ky = \lambda y$ $\lambda, y \Rightarrow$ eigenvalue and eigenvector

$-\frac{d^2y}{dx^2} = \lambda y$ $\lambda, y(x) \Rightarrow$ "eigenvalue" and "eigenfunction"

$-\frac{d^2y}{dx^2} = \lambda y$ $\begin{cases} y = \sin wx & \lambda = w^2 \\ y = \cos wx & \lambda = -w^2 \\ y = e^{-iwx} = \cos wx + i\sin wx & \lambda = -w^2 \end{cases}$

$y(0) = 0$, $y(1) = 0$ $\left\{ \begin{array}{ll} y(0) = 0 & \text{cos's are gone} \\ y(1) = \sin w = 0 & w = k\pi \end{array} \right]$ $y_1 = \sin\pi x \quad \lambda_1 = -\pi^2$, $y_2 = \sin 2\pi x \quad \lambda_2 = -4\pi^2$

$\Downarrow$

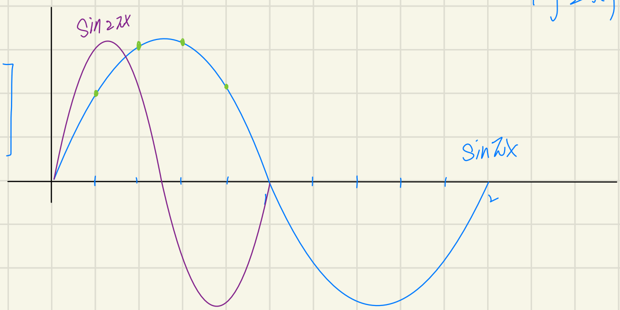
$K \begin{cases} y_1 = \sin\pi x & \lambda = \pi^2 \\ y_2 = \sin 2\pi x & \lambda_2 = 4\pi^2 \end{cases}$

$y'(0) = 0$, $y(1) = 0$ $\begin{cases} y'(0) = 0 & \text{sin's are gone} \\ y(1) = \cos w = 0 & w = (k + \frac{1}{2}\pi) \end{cases}$

$Ky = \lambda y$

$y_1 = \begin{bmatrix} \sin(\pi \cdot \frac{1}{5}) \\ \sin(\pi \cdot \frac{2}{5}) \\ \sin(\pi \cdot \frac{3}{5}) \\ \sin(\pi \cdot \frac{4}{5}) \end{bmatrix}$

$K = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$



$y_2 = \begin{bmatrix} \sin(2\pi \cdot \frac{1}{5}) \\ \sin(2\pi \cdot \frac{2}{5}) \\ \sin(2\pi \cdot \frac{3}{5}) \\ \sin(2\pi \cdot \frac{4}{5}) \end{bmatrix}$

$y_5 = \begin{bmatrix} \sin(5\pi \cdot \frac{1}{5}) \\ \sin(5\pi \cdot \frac{2}{5}) \\ \vdots \end{bmatrix} = \vec{0}$

$$Ky = \lambda y$$

$$y_1 = \begin{bmatrix} \sin\left(\pi \cdot \frac{1}{5}\right) \\ \sin\left(\pi \cdot \frac{2}{5}\right) \\ \sin\left(\pi \cdot \frac{3}{5}\right) \\ \sin\left(\pi \cdot \frac{4}{5}\right) \end{bmatrix} \rightarrow \sin \pi x$$

$$y_2 = \begin{bmatrix} \sin\left(2\pi \cdot \frac{1}{5}\right) \\ \sin\left(2\pi \cdot \frac{2}{5}\right) \\ \sin\left(2\pi \cdot \frac{3}{5}\right) \\ \sin\left(2\pi \cdot \frac{4}{5}\right) \end{bmatrix} \rightarrow \sin 2\pi x$$

$$\vdots \qquad\qquad \vdots$$

orthogonal vectors

orthogonal functions

$$\int_0^1 \sin \pi x \, \sin 2\pi x \, dx = 0$$