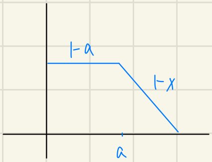
T^{-1}:

$TT^{-1} = I$



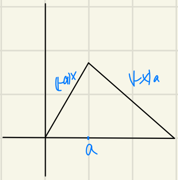
$$T^{-1} = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 4 & 3 & 2 & 1 \\ 3 & 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$(0 \quad 0 \quad 0 \quad 0 \quad 0)$

not sparse

free-fixed

K^{-1}:

$KK^{-1} = I$



$(0 \quad 0 \quad 0 \quad 0)$
$$K^{-1} = 1/5 \begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 6 & 4 & 2 \\ 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$
$(0 \quad 0 \quad 0 \quad 0)$

fixed-fixed

3 ↓ $u'(x) = 3$
6 ↓ $u'(x) = -2$
4
2

$u''(x) = 5$

Eigenvalue

$Ay = \lambda y$ for $\frac{du}{dt} = Au$:

$u(t) = c \cdot e^{\lambda t} \cdot y$ where (λ, y) is a eigenvalue-eigenvector pair of A

$e^{\lambda t} = e^{(a+bi)t} = e^{at}(e^{bit}) = e^{at} \cdot (\cos bt + i \sin bt)$
$\begin{cases} \text{real}(\lambda) < 0 & \text{decay / stable} \\ \text{real}(\lambda) > 0 & \text{growth / unstable} \\ \text{real}(\lambda) = 0 & \text{oscillating} \end{cases}$

eg. $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ rotate by $90°$ A^2 has eigenvalue -1

Differential equations

a differential system $\begin{cases} \frac{du}{dt} = Au \\ u(0) = \end{cases}$

eg. $\begin{cases} \frac{du_1}{dt} = -u_1 + 2u_2 \\ \frac{du_2}{dt} = u_1 - 2u_2 \end{cases}$ $A = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$

$Ay_i = \lambda y_i$

$u(0) = c_1 y_1 + c_2 y_2 \cdots c_n y_n$

$u(t) = c_1 \cdot e^{\lambda_1 t} y_1 + \cdots + c_n \cdot e^{\lambda_n t} y_n$

$\frac{du(t)}{dt} = \lambda_1 c_1 e^{\lambda_1 t} y_1 \cdots \lambda_n c_n e^{\lambda_n t} y_n$
$= A \cdot u(t)$

if λ_i has geometric multiplicity 1 but algebric multiplicity, (18.03)
we complete $u(x)$ with $\lambda_i \cdot c_i e^{\lambda_i t} y_i + \lambda_i c_i t \cdot e^{\lambda_i t} y_i$