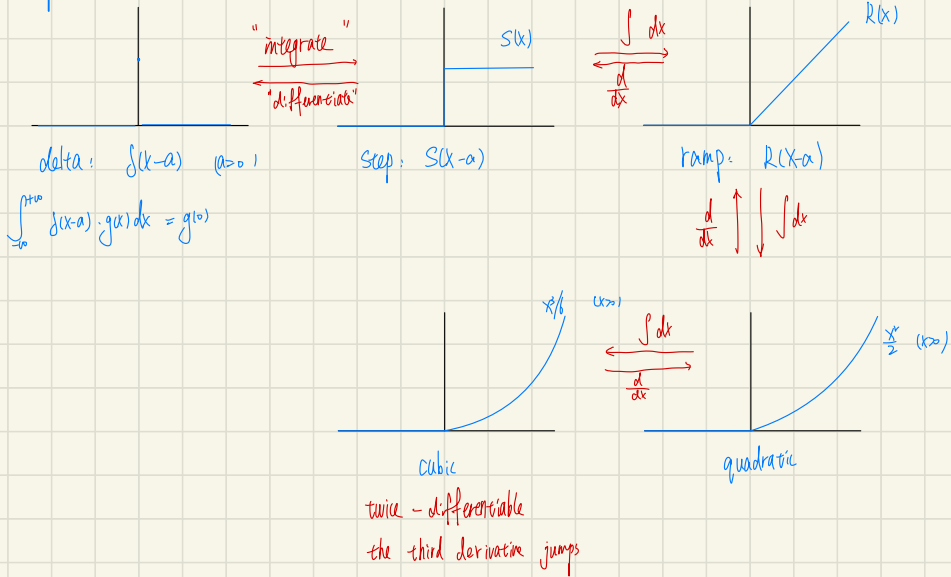




Delta function



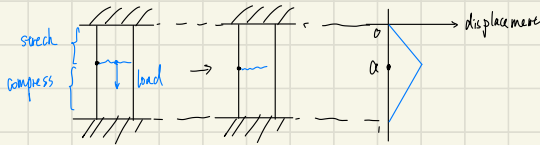
Bar displacement

fixed fixed

$$\begin{cases} -\frac{d^2 u}{dx^2} = \delta(x-a) \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$

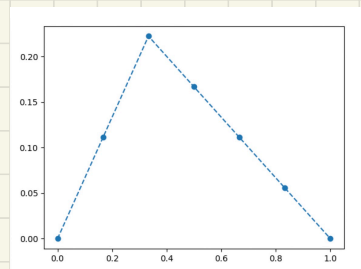
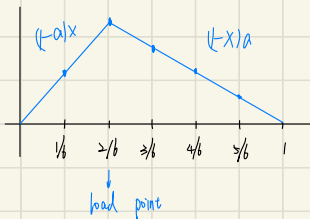
$$u(x) = -R(x-a) + Cx + D$$

$$= -R(x-a) + (1-a)x$$



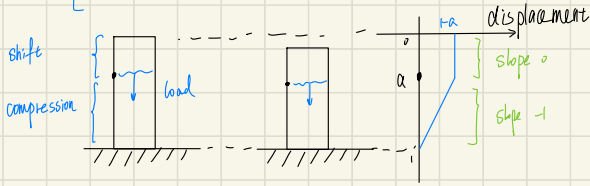
$$\frac{1}{dx^2} K u = \delta(x-1)$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



free
fixed

$$\begin{cases} -\frac{d^2 u}{dx^2} = f(x-a) \\ u(0) = 0 \\ u(1) = 0 \end{cases} \quad u(x) = -R(x-a) + Cx + D$$



$$\begin{cases} \frac{1}{\Delta x^2} k u = f(x-a) \\ u(1) = 0 \\ \frac{u_1 - u_0}{\Delta x} = 0 \end{cases}$$

$$\frac{1}{\Delta x^2} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$u_0 = 0$

