$Au=b$ $\begin{cases} ① & u = A/b \\ ② & \text{sparse: reorder rows} \\ ③ & \text{big: conjugate gradient} \end{cases}$

solving $Au=b$ does not compute A^{-1} eg. A is tridiagonal, A^{-1} is dense

· $A \backslash I$

$A \backslash I$ gives A^{-1}

$$A \begin{bmatrix} | & | & | \\ u_1 & u_2 & u_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

an impulse in the first component

response to impulse in the first component

find inverse:

$$\begin{bmatrix} A & I \end{bmatrix} \xrightarrow{\text{elimination}} \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

· LU

free-fixed

$$T = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \underset{\substack{r_2 + 1 \cdot r_1 \\ r_3 + 0 r_1}}{\overset{\substack{r_2 - 1 \cdot r_1 \\ r_3 - 0 \cdot r_1}}{\rightleftarrows}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \underset{r_3 + 1 \cdot r_2}{\overset{r_3 - r_2}{\rightleftarrows}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} — T_1 — \\ — T_2 — \\ — T_3 — \end{bmatrix}$$

$A = LU$

free-free

$$C = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \underset{\substack{r_2 + r_1 \\ r_3 + 0 r_1}}{\overset{\substack{r_3 - 0 r_1 \\ r_2 - r_1}}{\rightleftarrows}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \underset{r_3 + 1 r_2}{\overset{r_3 - 1 r_2}{\rightleftarrows}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

0 pivot → singular

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} — C_1 — \\ — C_2 — \\ — C_3 — \end{bmatrix}$$

fixed-fixed

$$K = \begin{bmatrix}2&-1&0\\-1&2&-1\\0&-1&2\end{bmatrix} \xrightarrow[r_2+\frac{1}{2}r_1]{r_2-\frac{1}{2}r_1} \begin{bmatrix}2&-1&0\\0&\frac{3}{2}&-1\\0&-1&2\end{bmatrix} \xrightarrow[r_3+\frac{2}{3}r_2]{r_3-\frac{2}{3}r_2} \begin{bmatrix}2&-1&0\\0&\frac{3}{2}&-1\\0&0&\frac{4}{3}\end{bmatrix} = \begin{bmatrix}2&&\\&\frac{3}{2}&\\&&\frac{4}{3}\end{bmatrix}\begin{bmatrix}1&-\frac{1}{2}&0\\0&1&-\frac{2}{3}\\0&0&1\end{bmatrix}$$

U

$$\begin{bmatrix}2&-1&0\\0&\frac{3}{2}&-1\\0&0&\frac{4}{3}\end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\-\frac{1}{2}&1&0\\0&-\frac{2}{3}&1\end{bmatrix}\begin{bmatrix}- & k_1 & -\\- & k_2 & -\\- & k_3 & -\end{bmatrix}$$

$$\underbrace{L\begin{bmatrix}1&0&0\\-\frac{1}{2}&1&0\\0&-\frac{2}{3}&1\end{bmatrix}}_{\text{brings back symmetry}}$$

$A = LDL^T$

if A is symmetric

$$A\begin{bmatrix}\cdot & \cdot & \end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

A^{-1}

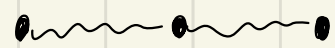
response in component 1 to impulse in component 2

response in component 2 to impulse in component 1

if the system is symmetric these 2 are the same

↓

A^{-1} is symmetric if A is symmetric



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