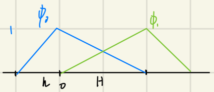
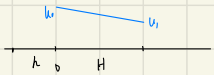
· Element matrices



not evenly spaced interval



$u(x) = u_0\phi_0(x) + u_1\phi_1(x)$

weak form: $\int_0^1 c(x) \frac{du}{dx}\frac{dv}{dx}\, dx = \int_0^1 f(x)\, v(x)\, dx \quad \forall v(x)$

$$\int_0^1 c(x)\left(u_0\phi_0'(x) + \cdots u_n\phi_n'(x)\right)\frac{dv_i}{dx}\cdot dx = \int_0^1 f(x)\, v_i(x)\, dx$$

$$\int_0^1 c(x)\left(u_0\phi_0'(x) + u_1\phi_1'(x)\right)\frac{d\phi_i}{dx}\, dx \Rightarrow Ku$$

$$\int_0^1 c(x)\left(u_0\phi_0'(x) + u_1\phi_1'(x)\right)\left(u_0\phi_0'(x) + u_1\phi_1'(x)\right) dx \Rightarrow u^T K u$$

$$\int_0^1 c(x)\left(\frac{du}{dx}\right)^2 dx = u^T K u$$

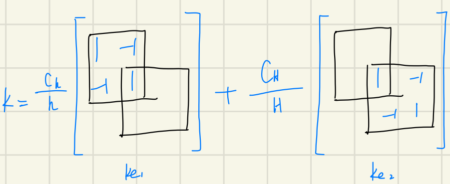
global K where $K_{ij} = \int_0^1 c(x)\frac{d\phi_i}{dx}\frac{d\phi_j}{dx}\, dx$

instead integrating $\phi_i' \cdot \phi_j'$ on the whole interval
integrate all ϕ_i's an element interval

$$\int_0^H c(x)\left(\frac{du}{dx}\right)^2 dx = u^T K_e u$$

$$\int_0^H \left(\frac{u_1 - u_0}{H}\right)^2 dx = \frac{1}{H}(u_1-u_0)^2 \cdot \int_0^H c(x)\,dx \rightarrow [u_0 \ \ u_1] \int_0^H c(x)dx\ \frac{1}{H}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_0 \\ u_1 \end{bmatrix}$$

$$\frac{c}{H}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \ K_e \quad \left(\text{take } c = c(H/2)\right)$$



simple idea in 1D, key idea in 2D

- 4th order bending equation

a very thin beam

fix-free beam (cantilever)

displacement $u(x)$ + boundary condition on u: $u(0)=0$, $u'(0)=0$ } fixed end

$\downarrow A = \frac{d^2}{dx^2}$

curvature e

$e = \frac{u''}{\sqrt{1+(u')^2}^3}$

small curvature, $(u')^2$ small compared to 1

$\xrightarrow{C}$ bending stiffness

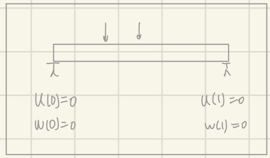
equilibrium equation $w'' = f(x)$

$\uparrow A^T = \frac{d^2}{dx^2}$ + boundary condition on w

bending moment $w=0$ $w'=0$ } free end

$w(x) = c(x) \cdot e(x)$

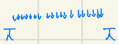
equivalent of Hooke's law



$$\frac{d^2}{dx^2}\left(c(x) \cdot e(x)\right) = \frac{d^2}{dx^2}\left(c(x)\frac{d^2u}{dx^2}\right) = f(x)$$

eg. when $c(x)=1$ $f(x)=1$ $u^{(4)}=1$

$$\begin{cases} u^{(4)} = 1 \\ u(0) = 0 \\ u''(0) = w(0) = 0 \\ u(1) = 0 \\ u''(1) = w(1) = 0 \end{cases}$$

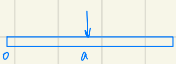


simply supported

uniform load

$u(x) = \frac{1}{24}x^4 + a + bx + cx^2 + dx^3$

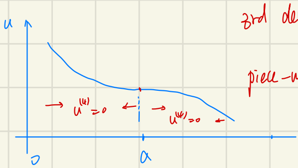
eg. when $c(x)=1$ $f(x) = \delta(x-a)$



$\delta(x-a)$ ↓ step ↓ ramp ↓ quadratic ↓ cubic $u_p = x^3/6$

$u(x) = x^3/6 + C_1 + C_2x + C_3x^2$

3rd derivative jumps



3rd derivative jumps → piece-wise cubic

1st derivative jumps → piece-wise linear

u(x) is a cubic spline