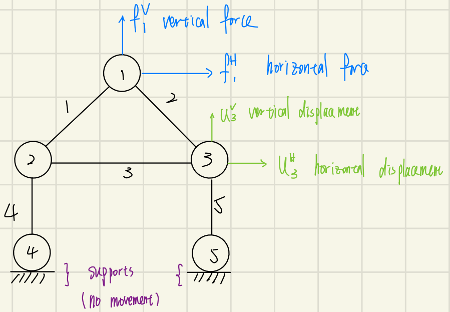


$m = 5$ bars

$N = 5$ nodes (pin points)

$n = 6$ (2 displacements for node 1, 2, 3)

each node has 2 forces and 2 displacements

$$\begin{tikzcd} u \in \mathbb{R}^6 \arrow[d, "A \in \mathbb{R}^{5\times 6}"'] & f \in \mathbb{R}^6 \arrow[d] \\ e \in \mathbb{R}^5 \arrow[r, "C \in \mathbb{R}^{5\times5}"'] & w \in \mathbb{R}^5 \end{tikzcd}$$

e: bar stretching

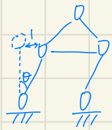
$A \in \mathbb{R}^{5\times 6}$

mechanism

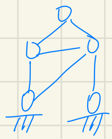
$\exists\, u \neq 0$ s.t. $Au = 0$

(nodes movement that does not stretch bars)

A^TCA is singular



$u = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ (small θ) $\longrightarrow$

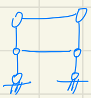


add an edge

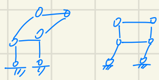
$A \in \mathbb{R}^{6\times 6}$

hopefully A is invertible

e.g.



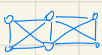
mechanism $= 2$



mechanisms

(not enough bars)

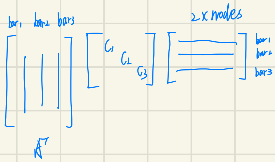
e.g.

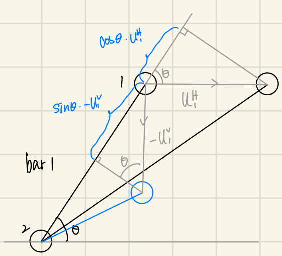


$Au = 0$ rigid motion $\begin{cases} \text{vertical movement} \\ \text{horizontal movement} \\ \text{rotation} \end{cases}$

rigid motions

(not enough supports)



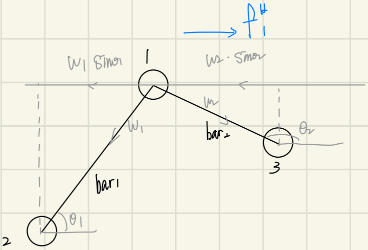


$$\begin{array}{cccccc} & u_1^H & u_1^V & u_2^H & u_2^V & \\ A[0,:] = [& \cos\theta, & \sin\theta, & -\cos\theta, & -\sin\theta, & 0 \cdots 0] \end{array}$$

$A[0,:] \cdot \begin{bmatrix} u_1^H \\ u_1^V \\ u_2^H \\ \vdots \end{bmatrix} =$ elongation in bar 1

elongation in bar: $e = Au$

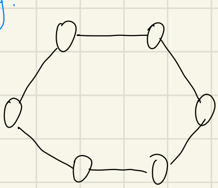
force in each bar: $w = \text{diag}(c) \cdot e$



$$A^T[0,:] = \begin{array}{c} \\ n_1 \\ \\ n_2 \\ \\ n_3 \\ \\ \end{array} \begin{array}{cc} \text{bar}_1 & \text{bar}_2 \\ \hline \cos\theta_1 & \cos\theta_2 \\ \sin\theta_1 & \sin\theta_2 \\ -\cos\theta_1 & \\ -\sin\theta_1 & \\ & -\cos\theta_2 \\ & -\sin\theta_2 \end{array} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad \begin{bmatrix} f_1^H \\ f_1^V \\ \\ \\ \\ \\ \end{bmatrix}$$

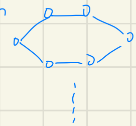
force balance: $f = A^T w$

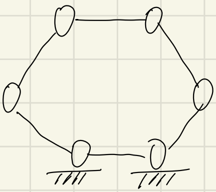
eg.



$A \in \mathbb{R}^{6\times 12}$

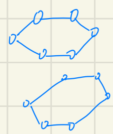
6 solutions to $Au=0$
- → 3 rigid motions
  - vertical movement
  - horizontal movement
  - rotation
- → 3 mechanism (independent)

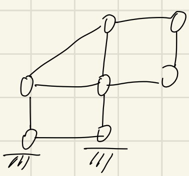




$A \in \mathbb{R}^{6\times 8}$

2 solutions to $Au=0$
- → 0 rigid motions
- → 2 mechanism





$A \in \mathbb{R}^{8\times 10}$

2 mechanism (independent)