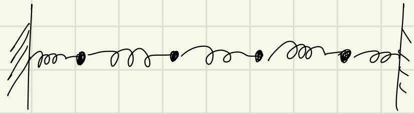
$$K = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

1. symmetric $K = K^T$
2. sparse (tridiagonal)
3. constant diagonal

K = toeplitz([2 -1 0 0])



4 masses attached by springs, with endpoints fixed

fixed - fixed

4. K invertible (row reduce)

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 1.5 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & 0 \\ 0 & 0 & -1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & 0 \\ 0 & 0 & 0 & \frac{5}{4} \end{bmatrix}$$

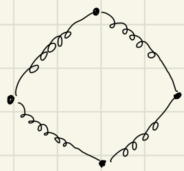
$\det(K) = 5$

$$C = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

"C" stands for circulant

come from a periodic problem

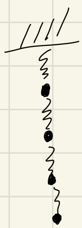
columns sum up to 0 : non-invertible



this system can move, with each mass has same displacement

so $\vec{1}$ is in the nullspace

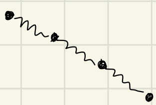
$$T = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$



free - fixed

invertible (not free to shift)

$$B = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$



free - free

non-invertible

4 matrices:

tridiagonal : each mass only connect to its neighbors

K and T: positive definite

C and B: positive semidefinite