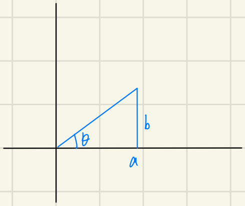


$a+bi = r\cos\theta + i\cdot r\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$= r\cdot e^{i\theta}$

$e^{i\theta} = \cos\theta + i\sin\theta$

$\theta \longrightarrow \boxed{e^{i\theta}} \longrightarrow$ complex

complex-valued function of a real variable

$u(t) + i\,v(t)$

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \begin{cases} ① \ e^{i\theta_1}\cdot e^{i\theta_2} = e^{i(\theta_1+\theta_2)} \\ ② \ \frac{d}{d\theta} e^{i\theta} = i\cdot e^{i\theta} \\ ③ \ \text{infinite series should work fine} \end{cases}$$

① $(\cos\theta_1 + i\sin\theta_1)\cdot(\cos\theta_2 + i\sin\theta_2)$

$= \cos\theta_1\cdot\cos\theta_2 - \sin\theta_1\sin\theta_2 + i(\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1)$

$= \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$

$e^{i\theta_1}\cdot e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$

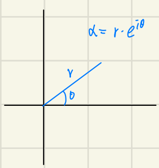
② $\frac{d}{d\theta}(\cos\theta + i\sin\theta) = -\sin\theta + i\cos\theta$

$= i(\cos\theta + i\sin\theta)$

$\frac{d}{d\theta} e^{i\theta} = i\cdot e^{i\theta}$ $\quad y(0) = e^{i\cdot 0} = \cos 0 + i\sin 0 = 1$

$e^{a+bi} = e^a\cdot e^{ib}$

polar form



r: modulus of α

θ: arguments of α (angle)

eg. $\int e^{-x}\cos x\,dx$

$= \int e^{-x}\,\text{real}(\cos x + i\sin x)\,dx$

$= \int e^{-x}\cdot \text{re}(e^{ix})\,dx$

$= \int \text{re}(e^{(-1+i)x})\,dx$

$= \text{re}\left[\int e^{(-1+i)x}\,dx\right]$

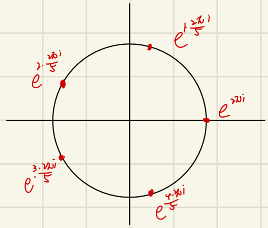
$= \text{re}\left[\frac{1}{-1+i} e^{(-1+i)x}\right]$

$= \text{re}\left[\frac{-1-i}{2}\cdot e^{-x}(\cos x + i\sin x)\right]$

$= \frac{e^{-x}}{2}(\sin x - \cos x)$

eg. $\sqrt[n]{1}$ (has n answers as complex numbers)

for $n=5$



$e^{2\pi i}=1$

$e^{i\theta_1}\cdot e^{i\theta_2} = \cos(\theta_1+\theta_2)\cdot \sin(\theta_1+\theta_2)$

$\left(e^{i\cdot\frac{2\pi}{5}}\right)^5 = e^{2\pi i} = 1$