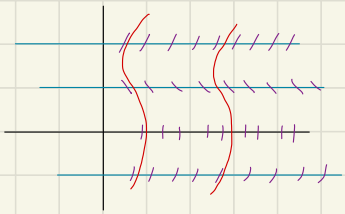
$dy/dt = f(y)$ Autonomous no independent variable on the rhs

can be solved by separating variables: how to get qualitative information without solving it



Slide an integral curve along x-axis giving another integral curve

- Critical points

y_0 s.t. $f(y_0)=0$ $y=y_0$ is a solution (integral curve is a horizontal line) ($dy/dt = f(y_0) = 0$ both sides are 0)

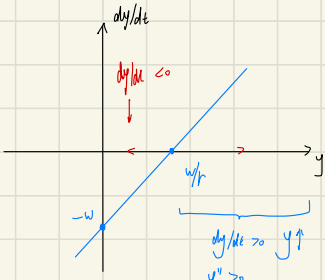
$y=y_0$ is an integral curve.
other curves are not allowed to cross them
$y=y_0$ is a barrier

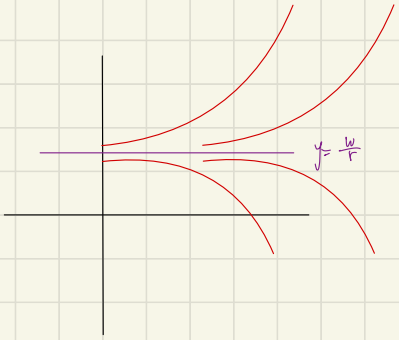
① find y_0 s.t. $f(y_0)=0$
② draw $f(y)$ when $dy/dt = f(y) > 0$ $y(t)\uparrow$

eg: y = money in bank
r = continuous interest rate
w = # of embezzlement

① critical points
$ry - w = 0$
$y = \frac{w}{r}$

$dy/dt = r \cdot y - w$





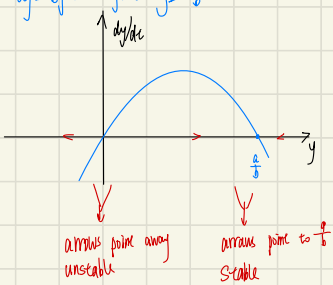
eg. logistic equation

y: population

$dy/dt = k$: the growth rate $\begin{cases} k \text{ constant} & \text{simple growth} \\ k = a - by \end{cases}$

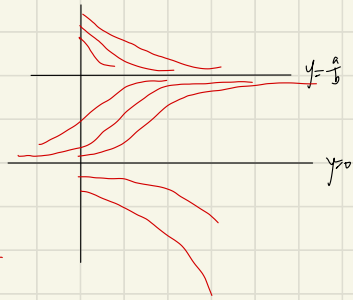
$dy/dt = ay - by^2$

$ay - by^2 = 0 \quad y = 0 \quad y = \frac{a}{b}$



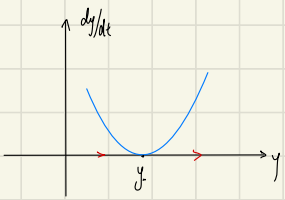
arrows point away unstable

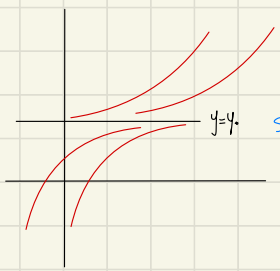
arrows point to $\frac{a}{b}$ Stable



Stable solution/critical point

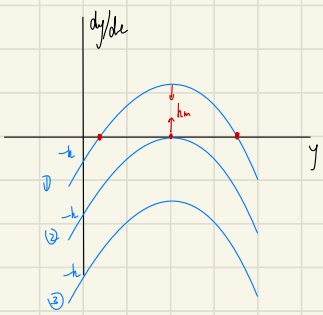
unstable solution/ critical point





semi-Stable

eg. $dy/dt = ay - by^2 - h$ h: harvest amount



hm: the maximum amount of harvesting