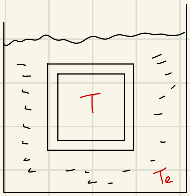
$a(x)y' + b(x)y = c(x)$ $\quad$ $c(x) = 0$: homogeneous

standard linear form $\quad y' + p(x)y = q(x)$

- Temperature – Concentration model

Conduction:

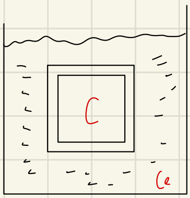


Newton cooling law

$$\begin{cases} \frac{dT}{dt} = k(T_e - T) \\ T(0) = T_0 \end{cases}$$

$k > 0$ is the conductivity

Diffusion



C: salt concentration inside

Ce: ··· outside

(semi-permeable membrane)

$$\frac{dc}{dt} = k(C_e - C)$$

$$\frac{dT}{dt} + \underbrace{k}_{\text{can change over time}} T = \underbrace{kT_e}_{\text{general function of } T_e}$$

- $y' + p(x)y = q(x)$

integrating factor: $u(x)$ $\quad$ want $\underbrace{u \cdot y' + pu\,y}_{(uy)'} = qu$ $\quad$ works if $u' = pu$

$u = e^{\int p(x)\,dx}$

no arbitrary constant since only 1 is needed

$$\frac{du}{dx} = p(x) \cdot u$$

$$\int \frac{1}{u}\,du = \int p(x)\,dx$$

1° put in standard form $y' + p(x) \cdot y = q(x)$

2° find integrating factor $u(x) = e^{\int p(x)\,dx}$

3° multiply both sides by $e^{\int p(x)\,dx}$

4° integrate

eg. $xy' - y = x^3$

① $y' + \left(-\frac{1}{x}\right)y = x^2$

② $u(x) = \exp\left(\int -\frac{1}{x}dx\right) = e^{-\ln x} = \frac{1}{x}$

③ $\frac{1}{x}y' - \frac{1}{x^2}y = x$

$\left(\frac{1}{x}y\right)' = x$

④ $\frac{1}{x}y = \frac{x^2}{2} + C$

$y = \frac{x^3}{2} + Cx$

eg. $\begin{cases} (1+\cos x)y' - \sin x \cdot y = 2x \\ y(0) = 1 \end{cases}$

① $y' - \frac{\sin x}{1+\cos x}y = \frac{2x}{1+\cos x}$

② $u(x) = \exp\left(\int -\frac{\sin x}{1+\cos x}dx\right) = e^{\ln(1+\cos x)} = 1+\cos x$

$y'(1+\cos x) - \sin x\, y = 2x$

$[y(1+\cos x)]' = 2x$

④ $y(1+\cos x) = x^2 + C$

$y = \frac{1}{1+\cos x}\cdot(x^2 + C)$ $\qquad y(0)=1 \Rightarrow C=2$

· **Interpretation of conduction model**

$\frac{dT}{dt} = k(T_e(t) - T)$

$\frac{dT}{dt} + kT = kT_e(t)$

1° $u(x) = e^{\int k\,dt} = e^{kt}$

2° $e^{kt}\cdot T' + k\cdot e^{kt}T = kT_e(t)\cdot e^{kt}$

$(e^{kt}\cdot T)' = kT_e(t)\cdot e^{kt}$

3° $e^{kt}\cdot T = k\cdot\int T_e(t)\cdot e^{kt}dt + C$

$T = e^{-kt}\cdot\int k\cdot T_e(t)\cdot e^{kt}dt + C\cdot e^{-kt}$

when $T(0) = T_0$

$$T(t) = \underbrace{e^{-kt}\cdot\int_0^t k\cdot T_e(v)\,e^{kv}dv}_{\text{Steady-state Solution}} + \underbrace{T_0\cdot e^{-kt}}_{\text{disappear as } t\to+\infty \text{ transient}} \qquad (k>0)$$