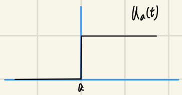
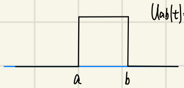
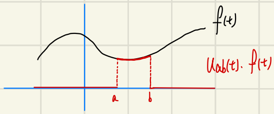
$u(t) := u_0(t)$



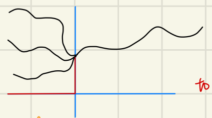


$u_{ab}(t) = u_a(t) - u_b(t)$



$$(\mathcal{L}u)(s) = \int_0^{+\infty} e^{-st} u(t)\, dt = \int_0^{+\infty} e^{-st} \cdot 1\, dt = 1/s$$

$$\begin{cases} f(t) \xrightarrow{\mathcal{L}} F(s) \\ F(s) \xrightarrow{\mathcal{L}^{-1}} f(t) \cdot u(t) \end{cases}$$



all have the same L.T

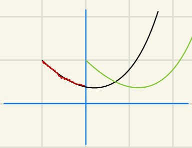
to have a unique $\mathcal{L}^{-1}$, make $f(t) = 0$ for $t < 0$

starting now and going on in the future

if don't have to know about the past, it's a Laplace Transform

if have to know about the past, it's a Fourier Transform

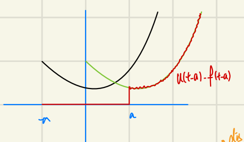
want a formular $\mathcal{L}(f(t-a))$ in terms of $\mathcal{L}f$ $(a > 0)$ no such a formula



this piece is not used for $\mathcal{L}f$

but needed for $\mathcal{L}(f(t-a))$

Right formula



$$\int_0^{+\infty} u(t-a) \cdot f(t-a) \cdot e^{-st}\, dt$$
$$= \int_{-a}^{+\infty} u(x)\, f(x) \cdot e^{-s(x+a)}\, dx$$
$$= \int_0^{+\infty} f(x)\, e^{-s(x+a)}\, dx$$
$$= e^{-as} \int_0^{+\infty} f(x)\, e^{-sx}\, dx$$
$$= e^{-as} (\mathcal{L}f)(s)$$

$$\int_0^{+\infty} u(t-a)\, f(t)\, e^{-st}\, dt$$
$$= \int_{-a}^{+\infty} u(x)\, f(x+a)\, e^{-s(x+a)}\, dx$$
$$= e^{-as} \int_0^{+\infty} f(x+a)\, e^{-sx}\, dx$$
$$= e^{-as}\, \mathcal{L}(f(t+a))$$

$$u(t-a)\, f(t-a) \xrightarrow{\mathcal{L}} e^{-as} \cdot F(s)$$

discontinuity at $t=a$

$$u(t-a) \cdot f(t) \xrightarrow{\mathcal{L}} e^{-as}\, \mathcal{L}(f(t+a))$$

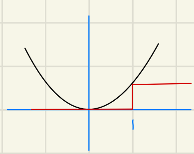
eg. $u_{ab}(t) = u_a(t) - u_b(t)$

$u_a(t) = u(t-a) = u(t-a) \cdot 1 \rightsquigarrow e^{-as} \frac{1}{s}$

$u_{ab}(t) \rightsquigarrow (e^{-as} - e^{-bs})/s$

eg. $u(t-1)\, t^2 = u(t-1) \cdot (t+1-1)^2 \quad f(t) = (t+1)^2$

$$e^{-s} \cdot \mathcal{L}((t+1)^2) = e^{-s} \mathcal{L}(t^2 + 2t + 1) = e^{-s}\left(\frac{2!}{s^3} + \frac{2 \cdot 1!}{s^2} + \frac{1}{s}\right)$$



eg $\mathcal{L}^{-1}\left(\frac{1+e^{-\pi s}}{s^2+1}\right)$: $\frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1}$

$\big\downarrow \mathcal{L}^{-1}$ $\quad\quad$ $\big\downarrow \mathcal{L}^{-1}$ $\quad\quad$ $\big\downarrow \mathcal{L}^{-1}$

$\underbrace{u(t)\sin(t)}_{t\geq 0} + \underbrace{u(t-\pi)\sin(t-\pi)}_{t\geq\pi}$ $\quad\quad$ $\sin t \cdot u(t)$ $\quad\quad$ $u(t-\pi)\sin(t-\pi)$

$\downarrow$

$$\begin{cases} \sin t & t\in[0,\pi] \\ \sin t + \sin(t-\pi) = \sin t - \sin t = 0 & t\in(\pi,+\infty] \end{cases}$$

$u(t-a)\cdot f(t) \rightsquigarrow e^{-as}\cdot \mathcal{L}f(t+a)$

$\mathcal{L}\sin t \to \frac{1}{s^2+1}$

$\mathcal{L}(f(t+\pi))$; $\rightsquigarrow \frac{1}{s^2+1}$

$f(t+\pi) = \sin t$

$f(t) = \sin(t-\pi)$