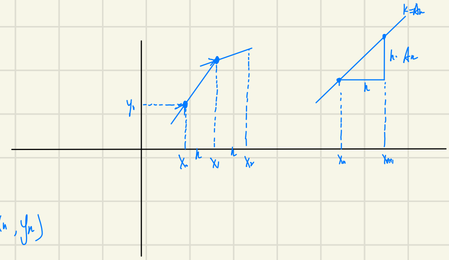
$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

- **Euler's method**



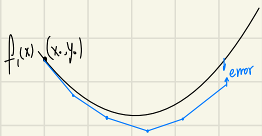
$x_{n+1} = x_n + h$

$y_{n+1} = y_n + h \cdot A_n$

$A_n = y'(x_n) = f(x_n, y_n)$

eg $\begin{cases} y' = x^2 - y^2 \\ y(0) = 1 \end{cases}$ $h = 0.1$

n	x_n	y_n	A_n	$h \cdot A_n$
0	0	1	-1	-0.1
1	0.1	0.9	-0.8	-0.08
2	0.2	0.82		



when the solution is convex, Euler too low

$y' = x^2 - y^2$

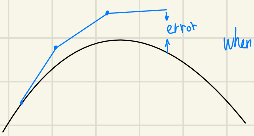
$y'' = 2x - 2y \cdot y'$

$y(0) = 1$

$y'(0) = -1$

$y''(0) = 2 \cdot 0 - 2 \cdot 1 \cdot -1 = 2$

$y(x)$ is convex at $(0,1)$



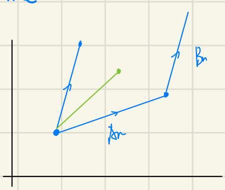
when the solution is concave, Euler too high

$f(x+\Delta x) = f(x) + \Delta x \cdot f'(x) + \frac{1}{2} \Delta x^2 f''(x)$

$\text{euler}(x+\Delta x) = f(x) + \Delta x \cdot f'(x)$

$e \sim O(h^2)$???

- **Better method** heun's method / modified Euler / RK2



$$\begin{cases} x_{n+1} = x_n + h \\ y_{n+1} = y_n + h \cdot \frac{(A_n + B_n)}{2} \\ \quad = y_n + h \cdot \left[A_n + f(x_n + h, y_n + h \cdot A_n) \right] \cdot \frac{1}{2} \end{cases}$$

RK4: 4 gradient evaluations. standard method (Runge-Kutta 4th order)

- **Pitfalls**

$y' = y^2$

$\Downarrow$

$y = \frac{1}{c - x}$

$y(0) = 1$

$y(2) = ?$