$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt \qquad f(t) \text{ period } 2\pi$$

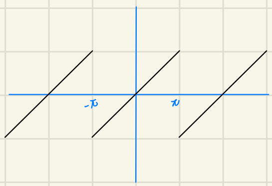
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \cos nt \, dt$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \sin nt \, dt$$

if $f(t) = f(-t)$, even function
$$\begin{cases} \text{the F.S only contains cos terms} \\ a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \cos nt \, dt = \frac{2}{\pi}\int_0^{\pi} f(t) \cos nt \, dt \end{cases}$$

if $-f(t) = f(-t)$ odd function
$$\begin{cases} \text{the F.S only contains sin terms} \\ b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \sin nt \, dt = \frac{2}{\pi}\int_0^{\pi} f(t) \sin nt \, dt \end{cases}$$

eg.



$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \sin nt \, dt$$

$$= \frac{2}{\pi}\int_0^{\pi} f(t) \sin nt \, dt = \frac{2}{\pi}\int_0^{\pi} t \sin nt \, dt$$

$$= \frac{2}{\pi}\left[\left.\frac{-t\cos nt}{n}\right|_0^{\pi} - \int_0^{\pi} \frac{-\cos nt}{n} \, dt \right] = \frac{2}{n}(-1)^{n+1}$$

Fourier Series model a function on an interval

Taylor Series model a function near a point

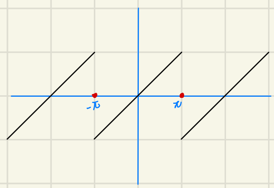
$$2\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} \sin nt$$

$$= 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t \cdots \right)$$

Theorem:

If $f(t)$ is continuous at t_0 then $f(t_0) = F.S.$ (pointwise converge)

if $f(t)$ has a jump discontinuity at t_0, F.S. converges at the midpoint of jump



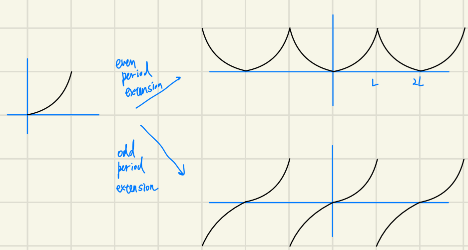
extension 1: period $2L$, not 2π

use natural function: $\cos \frac{n\pi}{L} t \qquad \sin \frac{n\pi}{L} t$

$$f(t) = \frac{a_0}{2} + \sum a_n \cos \frac{n\pi}{L} t + b_n \sin \frac{n\pi}{L} t$$

$$\begin{cases} a_n = \frac{1}{L}\int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} \, dt \\ b_n = \frac{1}{L}\int_{-L}^{L} f(t) \sin \frac{n\pi t}{L} \, dt \end{cases}$$

extension 2: $f(t)$ finite on $[0, L]$



period $2L$